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Conceptual Investigation of the Entropy Principle for Identification of Directives for Creation, Existence and Total Destruction of Order

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Abstract

Creation and existence of order are of philosophical significance and practical utility. This paper spells clear, succinct, yet comprehensive directives for order creation, existence and total destruction, in the form of nine exclusive directives established from the broken symmetry in the inter-convertibility of order and disorder stated by the *Entropy Principle* and the *Law of Maximum Entropy Production*. They form the guiding principles for order creation, stable existence and total destruction, at all scales. A representative mathematical model of order within disorder portrays these directives and spells the sustainability criterion for order existence. Since order is possibly stable only in chaotic systems, some of these directives have a probabilistic tone.

Nomenclature

F	symbol for field variable
k	Boltzmann's constant in Eq. (3) [J/K]
N_{thr}	threshold number of ordered sub-systems co-existing [-]
Q_{in}	heat energy input [J]
Q_{rej}	heat energy rejected [J]
ΔQ_{rev}	heat transferred reversibly [J]
S	entropy [J/K]
s	specific entropy [J/kg-K]
$\dot{s}$	time rate of increase of specific entropy [J/kg-K-s]
T	temperature [K]
t	time [s]
W	number of configurations a system can exist [-]
W_{out}	work (as output) [J]

Subscripts

c	creation of sustainable ordered sub-system within disorder
d	surrounding disorder with respect to sustainable ordered sub-systems
e	existence of sustainable ordered sub-system within disorder
o	sustainable ordered sub-system within disorder
sink	heat sink
source	heat source
thr	threshold
1,2,..,i,..N	ordered sub-systems 1,2,..i,..,N within disorder

Superscript

*	referring to state of total disorder

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1. Introduction and necessity

The creation and existence of *disorder* in nature is a spontaneous process that follows from "the Second Law of Thermodynamics ('the Entropy Principle')" according to which, the *entropy* of an isolated system that is not at equilibrium always increases, or equivalently the gradients of field variables within the system are minimised. Popular examples of such spontaneous processes that lead to dispersion and increase the disorder are diffusion in fluids and expansion of gases into vacuum (elaborated for instance by Lambert [1]).

1.1. *Background and motivation*

Every galaxy in the universe and every atom is an ordered structure. The origin of this observed order is sometimes attributed to an unknown external intervention; since, immediately after the Big Bang, the universe was in a completely disorganized state as though the entropy had been maximized. This change from disorder to order *appears* to be in violation of the Entropy Principle. It was stated by Planck [2] that a certain order prevails in nature; however, it was attributed to 'purposeful activity'. Einstein [3] described this order as a *miracle* that is strengthened by the development of knowledge. Bertalanffy [4] suspected that there might be other thermodynamic principle/s that would account for opportunistic ordering. Davies [5] stated that a highly ordered system that displays a great deal of complex organised activity, needs a lot of *information* to describe it; since, the complexity of a process is measured by the amount of information needed to predict its future behavior. It is stated in Layzer [6] that there exists a single universal law governing processes that dissipate order (the Entropy Principle), but *hypothesised* that order is generated by several hierarchically linked processes, including cosmic expansion and biological evolution.

The major revolution in the later half of the previous century is the understanding with an expanded view of thermodynamics that the spontaneous production of order from disorder is the expected consequence of basic laws. There is still considerable refinement necessary, however, to understand fully how this works. Creation and stable existence of order are not only of philosophical or conceptual significance, but also of practical and engineering utility, since invariably all human made constructive processes involve creation and maintenance of order from and within disorder, or control of disorder increase in a system through intervention of localised order. Since stable order is always sensed, manifested, preferred and desired, there is always a possibility of being overcome so as not to realise the existence of the broken symmetry between order and disorder and its implications. This may lead to several misconceptions about creation, existence and total destruction of order.

1.2. *Objectives and scope*

This paper first articulates the entropy principle, concepts of irreversibility, order and disorder, and then spells the directives as comprehensively as possible, in the form of nine exclusive punch-line Statements. Statements 1 and 2 enunciate order creation, Statements 3–5 discuss the sustained existence of order, Statement 6 spells the total destruction of order, Statement 7 collates creation, existence and total destruction of order, and Statement 8 discusses again order existence based on Statements 3 and 7. The final Statement 9 reveals the purpose for order creation and its fleeting existence. The sequencing of these statements is based on natural flow of conceptual analysis. They are established from the *Entropy Principle* and include the *Law of Maximum Entropy Production*. A representative model of an isolated system in which sustainable and stable order exists for a finite time, demonstrates these statements, and spells the sustainability criterion for stable order existence.

2. Irreversibility and entropy principle

The 2nd Law implies that physical systems tend to slide spontaneously, and due to irreversibilities towards a state of disorder. Hence, systems spontaneously tend to go from a less probable state in which they are relatively orderly organised (low entropy) to one that is more probable in which they are disorderly organised (high entropy). Non-spontaneous change needs external intervention, which is the basis for irreversible loss.

2.1. *Concept of irreversibility*

It is pertinent to articulate irreversibility, as it intricately relates the existence and operations of order within disorder. Irreversibility is a characteristic of all non-linear processes for which superposition is not valid, as a result of which, the system cannot return back to its' original state on its' own by simply reversing the output and input, which results in an irreversible loss (inability to reverse to previous state without external intervention). It is a propagating loss over several non-linear systems in the universe. There is no irreversible loss if reversibility is not attempted, thereby allowing spontaneous flow of processes in nature. Irreversible loss is realised whenever order co-exists with disorder, which necessitates a return to the original state or another specific (non-spontaneous) state for reducing entropy of order (for stable existence of order). It is the continuous cost paid to reverse back to the original state through real life non-linear processes. Irreversibility is a relative loss; since it is a loss for order and as First Law of Thermodynamics is always satisfied, it is a gain for the surrounding disorder. Since surrounding disorder is infinite relative to localised order, gain for disorder results in dispersion of the gain in disorder (distribution of finite to infinite), which is entropy generation.

All real world processes that entail reversibility encounter non-linearity, and one of the most practical examples is the popularly known thermodynamic irreversibility of the heat engine (when conversion of heat to work is desired), comprising of Q_{in} as input and W_{out} as output, and this system is represented by the classical equation:

$$Q_{\text{in}} = W_{\text{out}} + Q_{\text{rej}}. \tag{1}$$

The invalidity of superposition results from $Q_{\text{rej}} > 0$ (except for $T_{\text{sink}} = 0\,\text{K}$ and/or $T_{\text{source}} \to \infty$ for ideal Carnot engine, both of which are not possible), which necessitates infinite heat engines operating at monotonically decreasing cycle efficiency for complete conversion of Q_{rej} in to work, which is practically infeasible. Heat is disordered form of energy but work is ordered form of energy; hence, total conversion of heat to work would imply decreasing disorder and violation of the 2nd Law. Hence, some heat is rejected at lower temperature to increase the net disorder and satisfy the 2nd Law.

2.2. *Entropy principle*

The 2nd Law is also called the *Entropy Principle*, and it came in to existence after the First Law was established, but with the basic realisation that energy alone is not the motive force for change, and the perception that energy possesses quality in addition to quantity. The key insight is that whenever an energy distribution is out of equilibrium, a thermodynamic *force* exists that acts spontaneously to minimize this force. While the First Law expresses time-symmetry, the 2nd Law expresses that which changes and motivates the change, the fundamental time-asymmetry in all process. The word 'entropy' was coined to refer to the diminished gradients of field variables in a dissipation process, and as the classical signpost of natural change [7]. Entropy is conserved only in completely linear and hence reversible processes, and increases in all natural processes that are spontaneous, non-linear with respect to deviation from spontaneity and hence irreversible. The past, present, and future are indistinguishable with the First Law; but the one-way flow due to irreversibilities in natural processes introduced by the 2nd Law, brings importance to time.

Mathematically, the simplest definition of entropy transfer is given as (explained conceptually for instance in [8]),

$$\Delta S = \frac{\Delta Q_{\text{rev}}}{T}, \tag{2}$$

and the entropy generated due to irreversibilities is obtained by comparing the actual entropy change with the above entropy transfer (ref. for instance, [9]). The statistical interpretation of entropy is given by Boltzmann [10], who showed that the tendency of energy to disperse is equivalent to increase in entropy:

$$S = k \cdot \ln W, \tag{3}$$

thereby reducing the 2nd Law to a stochastic collision function, and generalising it as the Law of Disorder. As per this realisation, the initial non-equilibrium distributions become increasingly disordered leading to a final state of macroscopic uniformity and microscopic disorder. The increase in entropy due to increase in the number of accessible states as quantified by the above Boltzmann–Planck equation relating entropy and chaos, is conceptually illustrated in [11]. Boltzmann's interpretation of the Entropy Principle implies that a spontaneous change must be accompanied by an increase in accessible number of micro-states. It is just this Boltzmann's view that disorder increases monotonically, or simply stated, the world goes from order to disorder that is falsified by the spontaneous production of order from disorder (which as per Boltzmann's view is 'infinitely improbable'). This incomplete view of the 2nd Law led to the popular belief that the production of order required some outside agency or extra-physical operators [2,3].

The Entropy Principle presides as the ruling paradigm; it is accepted as the premier law of all sciences, and as the supreme metaphysical law of the entire universe [12]. It spells nature's dissymmetry in natural interchange of energy through dispersion. Conversely, it *globally* denies the spontaneous emergence of ordered structure. However, this does not imply that ordered structures cannot ever emerge, which is not directly addressed by the Entropy Principle. The Entropy Principle does not explain how complex ordered systems could have arisen from less ordered states and have maintained themselves in defiance of the tendency towards increasing disorder. The Entropy Principle also does not answer the question: *which out of available paths a system will take to accomplish net disorder increase*. This limitation is addressed by 'The Law of Maximum Entropy Production' which states that 'the system will select the path or assemblage of paths out of available paths that minimises the potential or maximizes the entropy at the fastest rate given the constraints' [13] (ref. also [14]). Swenson [15] states that "the active, end-directed, or intentional dynamics of living things, and evolution as a general process of dynamically ordered things that actively work to bring more order, follows directly from natural laws".

3. Concept of order vs. disorder

Entropy or the subjective term *disorder* is quantified by association with predictability of a statistical system {deduced from Eq. (3) [10]}, which is the probability of locating an identity in a statistical system in a particular state; this quantification is modelled for instance in [16]. The concept of entropy is intrinsically better suited for systems that are chaotic and dynamically unstable that can only be described probabilistically. In a qualitative sense, entropy increase results in dispersion and reduction of predictability, as per Eq. (3), which is an increase in level of system disorder. Conversely, ordered sub-systems are characterised by a low tendency towards dispersion relative to their surrounding disorder, through periodic coalescence and concentration of energy and matter for localised control of dispersion.

3.1. *System non-linearity*

In a perfectly ordered isolated system, all sub-systems and all the resulting processes are linear and hence reversible (linear in forward and reverse directions, unlike the inter-convertibility of heat and work); it is an ideal and hypothetical system. In practice, the behaviour of most systems is close to linear and reversible only when operating close to equilibrium (where equilibrium is the condition when all gradients of field variables are zero). An isolated system is highly disordered and the resulting processes are highly irreversible, when more sub-systems are highly non-linear, which is practically the case especially when operating far from equilibrium.

3.2. *Identity of ordered sub-system*

In conventional terms, order is viewed as neat, patterned and thus with an identity. Conversely, disorder is viewed as messy, random, chaotic, uncorrelated and incoherent and thus lacking an identity. This enunciation of order is based on human preference for order and identity. In nature, the preferred order is amongst the very few options available for spontaneous dispersion and the remaining numerous options without specific identity are referred as disorder. Hence, the probability of spontaneous dispersion as disorder well exceeds order formation and identity establishment. Nevertheless, ordered sub-systems existing within disorder maintain their respective unique identities during their fleeting existence by continuously reducing their specific entropies below a particular threshold. Surrounding disorder is attributed a single identity, but a given control volume within disorder does not have sustainable identity.

3.3. *Sustainability of gradient of field variables across ordered sub-system*

The existence of sustainable order with sustainable identity within disorder in an isolated system may be viewed as a result of the sustained existence of localised finite gradient of field variable/s within the ordered system and/or across the system boundary. An example that substantiates this view point is the human as a sustainable ordered system, temperature as field variable, and temperature gradient with respect to the surroundings is maintained across the system boundary. *Sustainability* refers to the stable existence of order in surrounding disorder.

4. Statements of directives and their implications

As concluded from the survey, one of the most interesting issues in the study of highly disordered systems that are unpredictable and hence termed chaotic, is whether or not the presence of chaos may actually produce and sustain ordered structures and patterns on a larger scale. As stated in the introduction, the major revolution in the later half of the previous century is the understanding with an expanded view of thermodynamics that the spontaneous production of order from disorder is the expected consequence of basic laws. There is still considerable refinement necessary, however, to understand fully how this works.

The statements proposed here are universal principles (for all scales ranging from cosmology to biology) for creation, stable existence and total destruction of order from and within disorder. They are identified based on their ability to universally integrate order generating, sustaining, and totally destroying processes without being defied, and are proved conceptually by deduction from the Entropy Principle and the Law of Maximum Entropy Production. The isolated system referred to in these statements is assumed to be large enough for all probabilistic events to occur, given a large but finite observation time interval. This assumption implies that the interactions within this isolated system are sufficiently large and the system is far from equilibrium, dynamically unstable, and chaos prevails. Hence, the probabilistic tone in some of these statements.

4.1. *Statement 1 (Creation of order)*

As the localised disequilibrium in an isolated system increases and exceeds a particular threshold, sustainable order is created within disorder whenever there is a 'chance'.

As per the Law of Maximum Entropy Production, in any process, "if ordered flow produces net entropy faster than disordered flow, and if the process minimises net gradients of field variables at the fastest rate, then the process can be *expected* to produce order whenever there is a *chance*" [17], as illustrated in Fig. 1. This *chance* indicates the broken symmetry in creation of sustainable order and net disorder, since net disorder is spontaneously (certainly) created, but creation of localised sustainable order is a *chance* (an uncertainty but a possibility). This *chance* is the probability that the sustainability criterion (to be illustrated later in Section 5) for co-existence of order within disorder is satisfied. The other aspect of broken symmetry results from the creation of order and disorder; since generation of disorder is not necessarily accompanied by creation of order; however, creation of order is always in conjunction with disorder (to satisfy the 2nd Law). This order generation process far away from equilibrium is the outcome of the purposeless operations of chaos. At a particular level of disorder, *collapse in to chaos* occurs, and stable sustainable ordered structures, referred as *autocatakinetic*, *self-organizing*, or *spontaneously ordered system* in literature (ref. e.g. [17]), may appear as singularities within disorder [18]. This threshold entropy bifurcates total disorder, and the co-existence of order and disorder. At this bifurcation point, the interactions between various states are not only sufficient for chaos to prevail but for chaos to reach its pinnacle so as to possibly create localised sustainable ordered structures. A practical example is the formation of ordered crystals from relatively disordered liquid. Thus, as per the Law of Maximum Entropy Production, the creation of sustainable order is not 'infinitely improbable' (as per the earlier interpretation of Boltzmann's hypothesis of the 2nd Law). Bertalanffy [4] showed that spontaneous order can appear in systems with energy flowing through them by their ability to build their order by dissipating potentials in their environments.

This statement also follows by deduction from the Entropy Principle as per which, *so long as a process is occurring in which more chaos is generated than is being destroyed, then the balance of energy may be withdrawn as coherent or orderly motion*. Alternatively, the state of more chaos can allow greater coherence locally, so long as greater dissipation has occurred elsewhere. The change from increasing disorder to increasing order is also illustrated in the system in the classical Boltzmann's Demon [8], and the entropy of the universe comprising of the two systems increases (ref. Fig. 2). The universe (isolated system) in Boltzmann's Demon is dynamically stable and there are no constraints that intervene in the process of increasing order of the system; hence, the increasing order of the system is deterministic. However, sustainable ordered sub-systems (referred in Statement 1) are created and exist within disorder only in a dynamically unstable isolated system that is far from equilibrium.

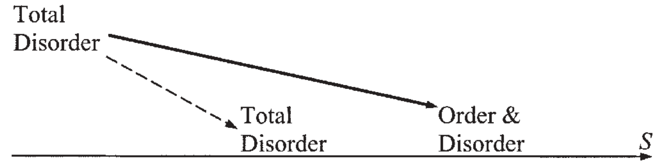

Fig. 1. Illustration of Statement 1.

The *chance* in Statement 1 is created by existing disorder probabilistically in a dynamically unstable system, and refers to constraint/s being removed or minimal threshold/s reached, or a coincidence of the two. For instance, evolutionary studies suggest that order was produced when the minimal threshold of an Earth cool enough so that it's oceans would not evaporate, and the threshold percentage of oxygen in the atmosphere were reached [19]. Though this order creating *chance* can be a low probability of the occurrence of events in a particular sequence at a given instant, the probability that these events occur in the required sequence approaches unity for large observation time.

Statement 1 is corroborated by Swenson [13,20]. For instance, Swenson [13] states that evolution on our planet can be viewed as an epistemic process by which the global system as a whole degrades the cosmic gradient at the fastest possible rate. Swenson [20] states that "opportunistic, self-accelerating structuring through the spontaneous emergence of increasingly specified levels of coherent reflexive states is an outcome of physical law".

4.2. *Statement 2 (Creation of order)*

In an isolated system, created order within disorder is a state of higher net disorder than the preceding state of total disorder.

This statement follows directly from the previous, and is illustrated in Figs. 1 & 2. It can also be directly deduced from the Entropy Principle. Since always net disorder must be created as per the Entropy Principle, sustainable order can be locally created only by increasing the disorder of the isolated system. The requirement for heat to be rejected at lower temperature during conversion of heat (disordered

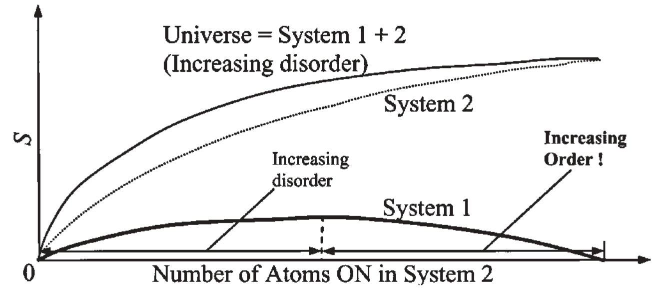

Fig. 2. Illustration of increasing order in Boltzmann's Demon (details in [8]).

energy) to work (ordered energy) is also spelt by this Statement.

It was suggested by Schroedinger [21] that sustainable orders like flames are permitted to exist away from equilibrium because they feed on 'negentropy' in their environments; which was later popularised by Prigogine [22] by the term 'dissipative structures'. Schroedinger's [21] important point was that as long as sustainable orders produce entropy at a sufficient rate to compensate for their own internal ordering or entropy reduction, then the balance equation of the 2nd law is not violated.

Order and disorder are different states, and a system in which both these states coexist dissipates gradients of field variables at a higher rate than the preceding state of the system having no ordered sub-systems at all but only disorder (which is the state of total disorder). Order within disorder is a more complex system (more non-linearity), and more information is required to describe it; hence, is a state of higher disorder than the previous state of total disorder. These interpretations hold for comparing the net disorder of two consecutive states of the system, one with total disorder and the next with created order within disorder.

4.3. *Statement 3 (Existence of order)*

In an isolated system, order must coexist with disorder, but disorder can exist alone as total disorder; or total order does not exist.

This statement indicates the broken symmetry in the existence of order and disorder. It follows from *Statements* 1 and 2 and hence from the Entropy Principle that order can only be created locally in an isolated system, and since the total disorder must increase, disorder must also exist together with order. However, since disorder can also increase without creation of order, disorder can exist alone. The term 'dissipative structures' is used to refer to ordered structures in Prigogine & Stengers [18], since they cannot exist independently of their environment (disorder). Low entropy values at which the system is dynamically stable is not referred as *total order* but as *low disorder*, since the low entropy of the system is not sustainable (as per the Entropy Principle). Sustainable order exists only relative to disorder, and is identified by its contrast with disorder.

4.4. *Statement 4 (Existence of order)*

In an isolated system, order and/or disorder must create more order and disorder for sustenance of created order; else, created order is converted in to disorder.

This statement indicates the broken symmetry in the sustenance of order and disorder. While disorder exists either by creating more disorder, or by creating order and disorder, or none of the two (under equilibrium), order exists only if it and/or the surrounding disorder create more order, and as a consequence more disorder also (as per Statement 2). It follows from the three exclusive choices available to order: (*a*) creation of additional order and disorder, (*b*) creation of only disorder, and (*c*) neither creation of order nor disorder. In options (*b*) and (*c*), the created order is converted in to disorder by the Entropy Principle. Option (*a*) remains the only viable option for order and/or surrounding disorder, so that the later created order generates more surrounding disorder and sustains the earlier created order. Swenson [23] presented an ecological physical view, which showed that—'purposive, creative behaviour is a consequence of natural law where order is produced such that order acts back upon order to produce more order'. An example of creation of order by earlier created order is the heat engine that converts heat (disordered energy) to work (ordered energy) that is also used by earlier created order for sustenance of its' disequilibrium with respect to surrounding disorder. In a philosophical sense, '*to live we must sustain our fleeting disequilibrium*' [8].

This statement also implies that a single ordered sub-system cannot exist alone and independently. Disorder alone cannot sustain created order; since statistically, the events that do not support the existence of order are far too many than those that support the existence of order. Hence, additional order must be created for the existence of previously created order, so that the resulting events and their sequence, sustain order.

4.5. *Statement 5 (Existence of order)*

In an isolated system, order can create more order only through disorder.

This statement indicates the broken symmetry in the creation of order and disorder; since more order is created only through disorder, but more disorder is generated through disorder alone or through the conversion of order to disorder. It is easily proved by negation. If order can create more order without any interaction with disorder, implies that the order is a localised isolated system. If order creates more order within itself, its order increases and entropy of this isolated system decreases. Thus, the assumption that this statement is false leads to a deduction which is against the Entropy Principle.

4.6. *Statement 6 (Total destruction of order)*

As the disorder of an isolated system containing order increases and exceeds another higher threshold, all order is destroyed in to total disorder again if there is a 'chance'.

Surrounding disorder can also increase disorder further by destroying all order. Order can generate more disorder also by destroying order. Whether order is created or destroyed by order and/or disorder is decided by *The Law of Maximum Entropy Production*, as per which, at a given instant, if more entropy is generated by destroying all existing orders in to disorder than the entropy generated by creation and existence of orders, then all existing orders will be destroyed in to disorder, if there is the *chance*. This *chance* is the probability that the sustainability criterion (to be illustrated later in Section 5) for co-existence of order within disorder is *not* satisfied.

Individual orders may also work against other existing orders when the system entropy has reached a value that is large enough for the existence of order within disorder to be unstable and hence unsustainable (to be described later in Section 6.1). This threshold entropy for total destruction of created order exceeds the threshold entropy for creation of order as per the Entropy Principle. This threshold entropy bifurcates the co-existence of order and disorder, and total disorder. The probability that event/s occur that lead to total destruction of all order, approaches unity as the time of order existence increases.

4.7. *Statement 7 (Creation, existence & total destruction of order)*

In an isolated system, creation, existence and total destruction of order within disorder is a sequential train with increasing entropy thresholds for order creation and total destruction.

Statement 6 spelt the existence of another threshold entropy level for destruction of all order in to total disorder. Once total disorder exists, Statement 1 follows, but the threshold entropy for order creation is now higher than the threshold entropy for order creation and total destruction in the previous sequence. Thus, creation, existence and total destruction of order, is a sequential train, and this sequence repeats along the signpost of increasing entropy, and hence, increasing thresholds of entropy for order creation and complete destruction (ref. Fig. 3). The sequential train of formation, growth and decay of coherent structures in turbulent flow that is chaotic [24], is practical phenomenon that exemplifies this proposed universal statement. In Fig. 3, the commencement is shown at a non-zero low entropy, since zero and negative entropy values of isolated systems are ruled out by the Third Law, enunciated for instance by Tamir [25] and illustrated in [26]. It is noteworthy that the very first compartment with low entropy values is also a state of total disorder, since order exists only relative to surrounding disorder and total order does not exist (ref. Statement 3). The boundaries between the consecutive compartments in Fig. 3 are the bifurcation zones, which are the threshold entropy values.

4.8. *Statement 8 (Existence of order)*

In an isolated system, disorder exists forever but existence of order is fleeting.

This statement indicates the broken symmetry in eternal existence, and follows from Statements 3 and 7, since disorder must exist whenever order exists, and from Statement 7, the isolated system either comprises of only disorder, or *order and disorder*. Fleeting order exists in the time interval between the thresholds of its creation and total destruction (ref. Fig. 3). This statement implies that the destruction of all order in to total disorder may be delayed.

4.9. *Statement 9 (Creation, existence & total destruction of order)*

In an isolated system, the role of creation and fleeting existence of order is to increase the system entropy at a faster rate than had order not existed.

This statement follows from an integration of the Entropy Principle and the Law of Maximum Entropy Production, and integrates the earlier eight statements. It gives the basis for creation and existence of sustainable and stable order. It is a result of the necessary propagating loss in the universe (isolated system) due to creation and existence of order that demands non-spontaneous and localised reversibilities in the midst of highly irreversible processes.

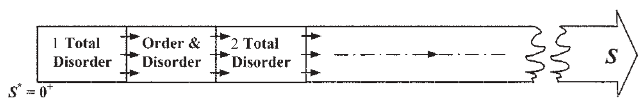

Fig. 3. Illustration of Statement 7.

Creation and fleeting existence of order is a higher order spontaneous process that results from a localised deviation from lower order spontaneity. More the deviation from lower order spontaneity, more is the net disorder generated as a consequence. The dramatic increase in the rate at which the potential is minimized in the Bénard cell experiment in the transition from disordered to ordered regime, is illustrated in [20] and also discussed in [27]. From this statement and Fig. 3, the following inequalities result: $\dot{s}_{d1} > \dot{s}^*_{d1}$, and $\dot{s}_{d1} > \dot{s}^*_{d2}$, where subscripts '1' and '2' refer to the compartment numbers in Fig. 3, and the superscript '$*$' refers to the state of total disorder. Thus, localised deviation from lower order spontaneity leading to higher order spontaneity not only satisfies the entropy principle as per which, $s_{d1} > s^*_{d1}$, but also results in faster entropy generation rate ($\dot{s}_{d1} > \dot{s}^*_{d1}$) since Law of Maximum Entropy Production is also satisfied.

5. Conceptual model exemplifying order in disorder

This section conceptually describes the dynamically unstable isolated system, in which chaos prevails. It illustrates the interaction between sustainable ordered sub-systems and surrounding disorder, for the creation, stable existence and total destruction of ordered sub-systems, thereby demonstrating the proposed nine Statements. It demonstrates how sustainable ordered sub-systems that are destined to exist only for finite time, increase the entropy of their surroundings at a faster rate than had they not existed. This model also illustrates the sustainability criterion for stability of order in disorder.

At a particular instant of time, consider the dynamically unstable Isolated System with N ordered sub-systems (ref. Fig. 4) with their specific entropies: $s_{o1}, s_{o2}, \ldots, s_{oN}$; respectively. These sub-systems are represented by dashed boxes to indicate that they must interact with their surroundings (disorder) for their sustenance, as per Statement 5 (Section 4.5). Since these sub-systems exist as sustainable and stable order within disorder, they maintain their unique identity during their existence. Hence, their specific entropy must be lower than a threshold ($s_{o,thr}$), i.e., for each i ordered sub-system,

$$s_{oi} \leq s_{o,thr}, \tag{4}$$

where $s_{o,thr} << s_d$. Since the differences in these thresholds are much lower than s_d, it is assumed for simplicity that this threshold value ($s_{o,thr}$) is the same for all ordered sub-systems. At $t = 0$, this isolated system is in a state of total disorder, i.e., no sustainable ordered sub-systems exist within the isolated system. At time instants: $t_{o1\_c}, t_{o2\_c}, \ldots, t_{oN\_c}$, sustainable ordered sub-systems: o1, o2, …, oN, respectively, are created and their respective identities are established. Again for simplicity, it is assumed in this model that as subsequent sustainable ordered sub-systems are created, all the previously created ordered sub-systems exist. The specific entropies of the surroundings at time, $t_{o1\_c}, t_{o2\_c}, \ldots, t_{oN\_c}$, are $s_d(t_{o1\_c})$, $s_d(t_{o2\_c}), \ldots,$ $s_d(t_{oN\_c})$, respectively. It follows that $s_{o1} \ll s_d(t_{o1\_c}), s_{o2} \ll s_d(t_{o2\_c}), \ldots, s_{oN} \ll s_d(t_{oN\_c})$; i.e.,

$$s_{oi} \ll s_d(t_{oi\_c}). \tag{5}$$

If sustainable ordered sub-systems are not created, which is the state of total disorder, the corresponding specific entropies of the isolated system in total disorder are: $s^*_d(t_{o1\_c}), s^*_d(t_{o2\_c}), \ldots, s^*_d(t_{oN\_c})$, at the same time instants: $t_{o1\_c}, t_{o2\_c}, \ldots, t_{oN\_c}$, respectively. It follows from Statement 2 (Section 4.2) that: $s_d(t_{o1\_c}) > s^*_d(t_{o1\_c})$, $s_d(t_{o2\_c}) > s^*_d(t_{o2\_c})$, …, $s_d(t_{oN\_c}) > s^*_d(t_{oN\_c})$; i.e.,

$$s_d(t_{oi\_c}) > s^*_d(t_{oi\_c}), \tag{6}$$

and from Statement 9 (Section 4.9), $\dot{s}_d(t_{oi\_c}) > \dot{s}^*_d(t_{oi\_c})$. These inequalities are due to increase in specific entropy of the surroundings w.r.t. sub-systems, due to creation and existence of sustainable ordered sub-systems. Thus, when sustainable ordered sub-system $o1$ is created,

$$s_d(t_{o1\_c}) = s^*_d(t_{o1\_c}) + \Delta s_{d,o1\_c}, \tag{6.1}$$

where, $\Delta s_{d,o1\_c}$ is the additional specific entropy increase in the surroundings w.r.t. sub-systems, due to irreversible processes that create ordered sub-system o1. Since order is created from the surrounding disorder, by conservation of mass, the mass of surrounding disorder decreases; however, this decrease is generally negligible, and this assumption is implicit in Eq. (6.1). Since a single ordered sub-system cannot be sustained as per Statement 4 (Section 4.4), additional sustainable ordered sub-systems are created for sustenance of created order sub-systems. When sustainable ordered sub-system o2 is created,

$$s_d(t_{o2\_c}) = s^*_d(t_{o2\_c}) + \Delta s_{d,o1\_c} + \Delta s_{d,o2\_c} + \Delta s_{d,o1\_e}(t_{o2\_c}). \tag{6.2}$$

The fleeting disequilibrium of ordered sub-system o1 with its surroundings is sustained through highly irreversible processes that generate net specific entropy, $\Delta s_{d,o1\_e}(t_{o2\_c})$, up to time instant t_{o2_c}. Similarly, when ordered sub-system oN is created,

$$s_d(t_{oN\_c}) = s^*_d(t_{oN\_c}) + \sum_{i=1}^{N} \Delta s_{d,oi\_c} + \sum_{i=1}^{N-1} \Delta s_{d,oi\_e}(t_{oN\_c}). \tag{6.3}$$

Equation (6.3) may be re-written as,

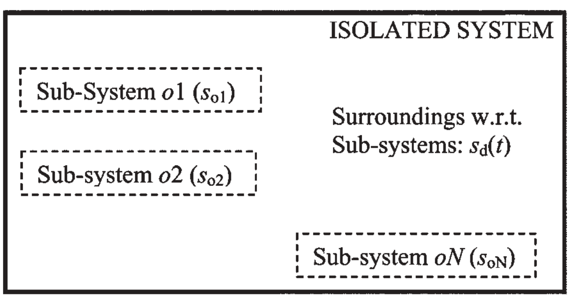

Fig. 4. Schematic illustration of sustainable order in disorder in an isolated system.

$$\frac{s^*_d(t_{oN\_c})}{s_d(t_{oN\_c})} = 1 - \frac{\sum_{i=1}^{N} \Delta s_{d,oi\_c}}{s_d(t_{oN\_c})} - \frac{\sum_{i=1}^{N-1} \Delta s_{d,oi\_e}(t_{oN\_c})}{s_d(t_{oN\_c})}, \tag{6.3.1}$$

where, the first entropy ratio on the right hand side is the ordered sub-systems creation ratio, and the second entropy ratio is the ordered sub-systems existence ratio. Now, for simplicity, it is assumed that all ordered sub-systems exist at the same specific entropy equal to their threshold, i.e.,

$$s_{o1} = s_{o2} = \cdots = s_{oN} = s_{o,thr}. \tag{7}$$

Hence, the difference between the specific entropies of sustainable ordered sub-systems and their surroundings increase with time, i.e.,

$$[s_d(t_{o1\_c}) - s_{o,thr}] < [s_d(t_{o2\_c}) - s_{o,thr}] < \cdots < [s_d(t_{oN\_c}) - s_{o,thr}]. \tag{8}$$

This increase in specific entropy difference between sustainable ordered sub-systems and their surroundings is equivalent to the gradient of field variable/s across the boundaries of sustainable ordered sub-systems increasing with time (discussed earlier in Section 3.3), i.e.,

$$\mathrm{Grad}[F(t_{o1\_c})] < \mathrm{Grad}[F(t_{o2\_c})] < \cdots < \mathrm{Grad}[F(t_{oN\_c})], \tag{9}$$

where these gradients are the magnitudes. However, a sustainable ordered sub-system is stable and exists provided the gradient of field variable/s is below a threshold $[\mathrm{Grad}(F)]_{thr}$. Alternatively, this also means that though ordered sub-system may be created since Law of Maximum Entropy Production is satisfied, but it may not exist because the gradient of field variable/s between this ordered sub-system and its' surrounding disorder is above this threshold $[\mathrm{Grad}(F)]_{thr}$. Thus, for existence of ordered sub-systems, the Law of Maximum Entropy Production and this sustainability criterion must be simultaneously satisfied. For simplicity it is assumed that this threshold value is the same for all N ordered sub-systems, since the differences in the threshold values are much smaller than spontaneously possible values of $\mathrm{Grad}(F)$. If $[\mathrm{Grad}(F)]_{thr}$ is reached immediately after the creation of ordered sub-system oN, i.e.,

$$\mathrm{Grad}[F(t^+_{oN\_c})] = [\mathrm{Grad}(F)]_{thr}, \tag{10}$$

then, at $t = t^+_{oN\_c}$, all N ordered sub-systems are unstable and hence unsustainable in their surroundings. Equation (10) is thus based on the sustainability criterion for the stable existence of ordered sub-systems within disorder, which in turn is based on Eq. (4). Once ordered sub-systems are unstable based on violation of the sustainability criterion, they are all destroyed in to total disorder once the Law of Maximum Entropy Production is satisfied, as per which more entropy should be generated by conversion of ordered sub-systems in to total disorder than by sustenance of ordered sub-systems within disorder [ref. Statement 6 (Section 4.6)]. The specific entropy of the isolated system in total disorder now is related as,

$$s_d^*(t_{oN\_c}^+) = s_d(t_{oN\_c}) + \sum_{i=1}^{N} \Delta s_{d,oi\to d}, \tag{11}$$

where $\sum_{i=1}^{N} \Delta s_{d,oi\to d}$ is the specific entropy increase in the isolated system due to irreversible processes that destroy all N ordered sub-systems in to disorder. The conversion to the state of total disorder may be delayed by increasing $s_{o,thr}$ and/or by increasing $[\text{Grad}(F)]_{thr}$. However, $s_{o,thr}$ and $[\text{Grad}(F)]_{thr}$ are not necessarily independent of each other, and increasing $s_{o,thr}$ may reduce $[\text{Grad}(F)]_{thr}$ and vice versa. In practice, different ordered sub-systems have different thresholds that are finite values; however, since the system net entropy is always destined to increase, the above conceptual implications based on the sequence of events still holds. It is possible that ordered sub-systems can delay the destruction of an individual ordered sub-system by increasing its '$[\text{Grad}(F)]_{thr,i}$'; but the irreversible processes that increase this individual threshold, increase the system entropy at a faster rate.

Combining Eqs. (6.3) and (11),

$$s_d^*(t_{oN\_c}^+) = s_d^*(t_{oN\_c}) + \sum_{i=1}^{N} \Delta s_{d,oi\_c} + \sum_{i=1}^{N-1} \Delta s_{d,oi\_e}(t_{oN\_c}) + \sum_{i=1}^{\mathrm{N}} \Delta s_{d,oi\to d}. \tag{11.1}$$

The $s_d^*(t_{oN\_c}^+)$ and $s_d^*(t_{oN\_c})$ are the specific entropies of the isolated system in two different states of total disorder. The $s_d^*(t_{oN\_c})$ is the specific entropy of the isolated system in total disorder at time instant t_{oN_c}, without sustainable ordered sub-systems existing earlier; and $s_d^*(t_{oN\_c}^+)$ is the specific entropy of the isolated system in total disorder at the same time instant, but in which ordered sub-systems have existed earlier. Thus, from Eqs. (6.3) and (11.1),

$$s_d^*(t_{oN\_c}^+) > s_d(t_{oN\_c}) > s_d^*(t_{oN\_c}). \tag{12}$$

6. Implications of directives based on conceptual model

The preceding model explicitly demonstrates that the creation and fleeting existence of ordered sub-systems, increases the entropy of the isolated system at a faster rate as compared to the entropy increase in a state of total disorder alone [Statement 9 (Section 4.9)]. Also, other Statements based on the Entropy Principle are illustrated in this model.

Order cannot be sustained in equilibrium with disorder, since order does not exist. However, order is unstable too far away from equilibrium with disorder (beyond the threshold). Thus, sustenance of order within disorder is the sustenance of the fleeting disequilibrium between order and disorder. This *sustainability* results from stability of order in disorder, and stability in turn is a result of one or more cyclic process in which order is continuously involved to maintain its' specific entropy level for maintaining its' identity within disorder. These cyclic processes oscillate the state of ordered sub-systems about the equilibrium (with disorder) state. Popular examples of such cyclic processes are the stability of the ordered solar system due to orbital motion of planets, and the cyclic process of breathing and metabolism that sustains human beings in their surroundings. This requirement to continuously reverse back to earlier state through non-linear processes is what leads to an irreversible loss. These cyclic processes involving ordered sub-systems increase the entropy of the surroundings at a faster rate.

6.1. *Benign and hostile role of order*

This model also exemplifies the role of additionally created order in hastening the approach to total disorder. While additionally created order sub-systems are necessary for the sustenance of other individual ordered sub-systems (ref. Statement 4, Section 4.4), their creation and existence increase the system entropy at a faster rate than had they not been created [ref. Eq. (6.3)]. Hence, creation and existence of additional ordered sub-systems tend to hasten the conversion to the state of total disorder. Ordered sub-systems enable earlier created ordered sub-systems to satisfy the inequality given by Eq. (4), by enabling them to reduce their entropy through cyclic processes; this benign role reduces $\text{Grad}[F(t)]$. In this benign role, ordered sub-systems work towards avoiding the *chance* (ref. Statement 6, Section 4.6) that can lead to total destruction as per the Law of Maximum Entropy Production. However, their creation and existence increase s_d, which increases $\text{Grad}[F(t)]$. This hostile role of ordered sub-systems to themselves takes them further away from equilibrium with their surroundings, and from Eqs. (8) and (9), the gradient of field variables between ordered sub-systems and their surroundings increase at a faster rate than had more ordered sub-systems not been created. Whether additionally created ordered sub-systems are benign or hostile to themselves is determined by $s_d(t)$. If $s_d(t)$ is not large and hence, $\text{Grad}[F(t)] \ll [\text{Grad}(F)]_{thr}$, ordered sub-systems are benign to themselves. Alternatively, if $s_d(t)$ is large and hence, $\text{Grad}[F(t)] \sim [\text{Grad}(F)]_{thr}$, ordered sub-systems are hostile to themselves, unstable, unsustainable and prone to total destruction.

It is noteworthy that creation and total destruction of order that is decided by the *Law of Maximum Entropy Production*, chooses the direction that maximises the entropy generation at a given instant of time. The instantaneously chosen direction need not necessarily maximise entropy generation thereafter. An illustration based on this model is the decision of total destruction of order once Eq. (10) is satisfied, which is based on *Law of Maximum Entropy Production* that decides between the choices of order destruction and sustenance at time instant t_{oN_c}. While at time t_{oN_c}, total destruction of order yields higher entropy generation than their sustenance, but subsequently the entropy increase is slower than had order existed (as per Statement 9).

The conceptual model and the implications of the earlier proposed statements are summarised by the flow-chart in Fig. 5. Throughout the flow in this chart, s_d or s_d^* monotonically increase, whether or not ordered sub-systems exist (s_d and s_d^* are associated with time due to irreversibilities).

6.2. *Implications based on alternative model*

In the above model, if it is assumed that the number of ordered sub-systems remains constant once a threshold value (N_{thr}) is reached, then once N_{thr} sub-systems are created, the number of ordered sub-systems created is the same as the number of ordered sub-systems destroyed. Individual ordered sub-systems may be destroyed because their $[\mathrm{Grad}(F)]_{thr}$ reduce significantly due to increase in their $s_{o,thr}$ with time and $s_d(t)$. In practice, N_{thr} is decided by the probability that $[\mathrm{Grad}(F)]_{thr}$ is exceeded for a particular ordered sub-system, due to inability of ordered sub-system/s to avoid high $\mathrm{Grad}(F)\{> [\mathrm{Grad}(F)]_{thr}\}$ created spontaneously at a given instant of time.

The essential difference in this alternative model is the additional entropy of destruction of individual ordered sub-systems ($\Delta s_{d,oi\to d}$) (instantaneous process), and the consequent subtraction of their existence entropy ($\Delta s_{d,oi-e}$) that is much larger due to integration over finite time. Since s_d is destined to increase though N_{thr} is fixed, the unstable condition of ordered sub-systems within disorder determined by Eq. (10) occurs later than in the previous model.

7. Summary and concluding remarks

(a) The proposed nine statements form the guiding principles for creation, existence and total destruction of order, at all scales. They are based on the broken symmetry in nature in the inter-convertibility of order and disorder, which is addressed by the *Entropy Principle*. Creation and destruction of order is also decided by the *Entropy Principle*, and the *Law of Maximum Entropy Production*, which decides between the options of creation of *order and disorder* or *disorder alone*, and destruction or maintenance of created order by creation of additional order, at a given time instant. The role of chaos in a dynamically unstable system that is necessary for sustainable order, gives some of these statements a probabilistic tone, and hence the reference to *chance* in these statements.

(b) The states of total disorder and co-existence of order in disorder are bifurcated by the two threshold entropies of creation of order in total disorder and total destruction of all order into total disorder.

(c) The fleeting existence of order is determined jointly by the *Law of Maximum Entropy Production*, and the *sustainability criterion* for existence of order within disorder. Sustainability of order within disorder is a result of one or more continuous cyclic process involving order.

(d) Creation and existence of ordered systems is a higher order spontaneous process that results from a localised deviation from lower order spontaneity.

(e) In conclusion: System non-linearity results in irreversibilities, irreversibilities create Disorder, increased Disorder creates Chaos, Chaos can create Sustainable Order, Creation and Existence of Order which is non-linear creates more Disorder, and then increased Disorder eventually Destroys all Order in to Total Disorder. This summary is illustrated by the block diagram in Fig. 6. Hence, existing order is neither a *miracle*, nor is it a *purposeful activity*, but a natural manifestation of the proposed guiding principles.

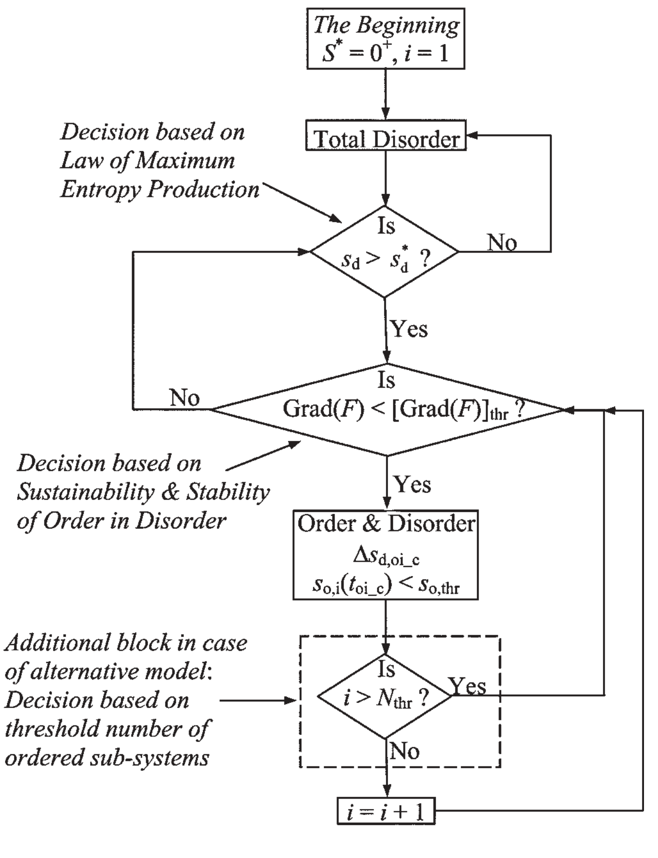

Fig. 5. Flowchart illustrating order creation, existence, and total destruction.

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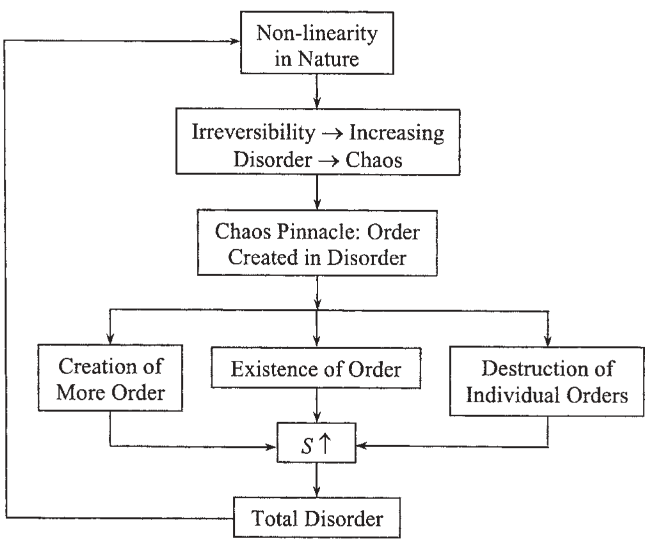

Fig. 6. Block diagram summarising the present conceptual investigation.

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